A decibel, which measures the relative intensity of sounds, has a logarithmic scale. Prolonged exposure to noise intensity exceeding 85 decibles can lead to hearing loss.

Definition and Properties of Base-10 Logarithms

To gain confidence in the meaning of logarithm, press LOG 3 on your calculator. You will get

$$\log 3 = 0.477121254\ldots$$

Then, without rounding, raise 10 to this power. You will get

$$10^{0.477121254\ldots} = 3$$

The powers of 10 have the normal properties of exponentiation. For instance,

$$15 = (3)(5) = (10^{0.4771\ldots})(10^{0.6989\ldots})$$

$$= 10^{0.4771\ldots+0.6989\ldots} \qquad \text{Add the exponents; keep the same base.}$$

$$= 10^{1.1760\ldots}$$

You can check by calculator that $10^{1.1760\ldots}$ really does equal 15.

From this example you can infer that logarithms have the same properties as exponents. This is not surprising, because logarithms *are* exponents. For instance,

$$\log(3 \cdot 5) = \log 3 + \log 5 \qquad \text{The log of a product equals the sum of the logs of the factors.}$$

From the values given earlier, you can also show that

$$\log \frac{15}{3} = \log 15 - \log 3 \qquad \text{The log of a quotient.}$$

This property is reasonable because you divide powers of equal bases by subtracting the exponents.

$$\frac{15}{3} = \frac{10^{1.1760\ldots}}{10^{0.4771\ldots}} = 10^{1.1760\ldots-0.4771} = 10^{0.6989\ldots} = 5$$

Because a power can be written as a product, you can find the logarithm of a power like this:

$$\log 3^4 = \log(3 \cdot 3 \cdot 3 \cdot 3) = \log 3 + \log 3 + \log 3 + \log 3$$

$$= 4 \log 3 \qquad \text{Combine like terms.}$$

The logarithm of a power equals the exponent of that power times the logarithm of the base. To verify this result, observe that $3^4 = 81$. Press 4 LOG 3 on your calculator, and get 1.9084…. Then press LOG 81. You get the same answer, 1.9084….